FX Skew from Market Quotes

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QuarcHome / 2026 / Version 1.0

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SUGGESTED CITATION

Kozlowski, C. (2026). *FX Skew from Market Quotes* (Version 1.0). QuarcHome.
quarchome.org/pdf/fx-skew-from-market-quotes.pdf

Abstract

What price would a euro-area exporter pay today for the right to sell one million euros for dollars in six months? An FX options screen may give a volatility level and two spreads, while the exporter needs a strike and a dollar premium. This article works from three synthetic quotes to those quantities. A call-minus-put risk reversal of −1.60 volatility points gives the euro put the higher implied volatility. Under the conventions specified below, the 25-delta put buys a floor of about 1.07533 dollars per euro for about 13,763 USD. A second contract that loses its protection if the euro touches a barrier raises a different question. Valuing it requires assumptions about the route the exchange rate takes to expiry.

Reading three quotes for one expiry

In plain English, the screen tells the exporter how dealers price protection near today's forward rate and how they price larger moves on either side. The first task is to identify the promise being priced. A 25-delta put label stands for a particular exchange rate at which the exporter may sell euros. A strangle quote stands for the combined cost of protection against a fall and a rise. Confusing a package price with an average of two volatilities changes the dollar answer, even if the same three numbers appear on both screens.

Let S be the spot exchange rate in domestic currency per unit of foreign currency. For EUR/USD quoted at 1.1200 dollars per euro, USD is domestic and EUR is foreign in the pricing formula. Here “domestic” identifies the unit of account, regardless of where the holder lives. A call on the euro gives its owner the right to buy euros at a fixed dollar price K, the strike. A put gives the right to sell. For one euro of notional, their dollar payoffs at expiry T are $\max(S_T - K, 0)$ and $\max(K - S_T, 0)$ respectively.

An *implied volatility* is the annualized volatility that reproduces an option premium in an agreed pricing formula. Writing it against strike for a single expiry gives a *smile*. Dealers can identify strikes by delta, a sensitivity of the option value to the exchange rate. The delta of a purchased euro call is positive and that of a purchased euro put is negative. A “25-delta put” here has delta −0.25. Its strike depends on the

volatility and the delta convention.

Quote	Definition for this article	What the quote measures
At the money (ATM), A	Volatility of the call and put at the strike where their deltas sum to zero	The level at the centre of the smile
25-delta risk reversal, R	Volatility of the +0.25 call minus volatility of the −0.25 put	How much more or less implied volatility the market assigns to the euro upside than to its downside
25-delta smile butterfly, B	Mean of those two wing volatilities minus A	How far the two wings sit above the centre on average

The wing options have different strikes. Their volatility spread allows a comparison on a common quoting scale even though the options buy protection against different moves. One *volatility point* is one percentage point of annualized volatility. A risk reversal of −1.60 points is a difference of −0.0160 in decimal volatility. It has no dollar units.

Kainth and Saravanamuttu (n.d., slide "European Implied Volatility Surface") describe both 25-delta and 10-delta quotes, giving five smile points when combined with ATM. Their slide writes the butterfly as the **sum** of wing volatilities less twice ATM. Its numerical value is $2B$ in the notation here. The 25-delta statistic also depends on where the two strikes fall. Recovering distributional moments such as kurtosis would require information about the rest of the smile, particularly its tails.

From a quote to a contract

Let r_d and r_f be constant, continuously compounded domestic and foreign rates. Take expiry and delivery to coincide, and express T in years from today. The forward rate for exchange at T and the domestic discount factor are

$$F = Se^{(r_d - r_f)T}, \qquad D_d = e^{-r_d T}.$$

The forward reflects the different interest earned by holding the two currencies. For example, buying $e^{-r_f T}$ euros today delivers one euro at expiry if invested at the foreign rate. Financing that purchase in dollars costs F dollars at expiry.

Write Φ for the standard normal cumulative distribution function and σ for an option's quoted volatility. In the lognormal FX pricing formula define

$$w = \sigma\sqrt{T}, \qquad d_1 = \frac{\log(F/K)}{w} + \frac{w}{2}, \qquad d_2 = d_1 - w.$$

For one unit of foreign notional the domestic premiums are

$$C = D_d[F\Phi(d_1) - K\Phi(d_2)], \qquad P = D_d[K\Phi(-d_2) - F\Phi(-d_1)].$$

Grabbe (1983, equations 3, 5-6 and 14-17) derives the forward relation, European put-call parity and the two option prices using domestic and foreign discount bonds. His journal article allows a more general interest-rate setting. Here the bond prices are exponential discount factors and the total variance for pricing a quoted leg is $\sigma^2 T$. One way to read the call formula is to split the exchange into two payments. The holder receives the euro on exercise and pays K dollars. The domestic pricing probability of exercise in this constant-volatility calculation is $\Phi(d_2)$. The euro receipt is especially valuable in states where its dollar price is high, which produces a different weight, $\Phi(d_1)$, on the euro side. This also explains why a 25-delta label cannot be read as a 25% probability of exercise.

We use **premium-unadjusted forward delta**. It is the change in the undiscounted option value per unit change in F, holding strike, volatility and discount factors fixed. The call delta is $\Phi(d_1)$ and the put delta is $\Phi(d_1)-1$. An unadjusted spot delta would multiply each by $e^{-r_f T}$. A premium-adjusted convention also accounts for the premium in the hedge quantity. Each choice can give a different strike for the same delta label (Reiswich and Wystup 2010, equations 3-12).

For the ATM straddle, buying a call and a put at the same strike and volatility gives a combined delta of $2\Phi(d_1)-1$. It vanishes when $d_1 = 0$, so $K_A = F\exp(A^2T/2)$. An ATM-forward quote would instead specify $K = F$. At positive volatility these strikes differ, even with identical rates and maturity.

For the wings, the mean volatility is $A+B$ and the call-minus-put difference is R. Solving for the two individual quotes gives

$$\sigma_C = A + B + R/2, \qquad \sigma_P = A + B - R/2.$$

Under this arithmetic butterfly convention the conversion is exact. To find a strike, use $d_1 = \Phi^{-1}(0.25)$ for the call or $d_1 = \Phi^{-1}(0.75)$ for the put. With each leg's own w, rearrangement gives $K = F\exp(w^2/2 - wd_1)$. The negative call quantile puts the call strike above the forward. The positive put quantile places the put strike below the forward for the inputs used here.

When the screen quotes a market strangle

A strangle buys a put at a lower strike and a call at a higher strike. Under the *market-strangle* convention described by Reiswich and Wystup (2010, equations 19-25 and 31), a spread M specifies a common volatility $A+M$. Use that single volatility to find both 25-delta strikes. Price both options at the same common volatility and add the premiums. This produces a dollar price for a particular two-leg contract.

When fitting a smile, keep those two contract strikes fixed and require their combined smile-priced premium to equal that dollar price. Separately, require the volatility difference at the smile's own 25-delta strikes to equal R, and the ATM volatility to equal A. This usually entails selecting an interpolation rule and solving for a parameter. The two sets of strikes can differ because delta changes with volatility. Substituting M for B in the arithmetic formula can therefore match the risk reversal yet miss the price of the quoted strangle.

Worked example in EUR/USD

All inputs are synthetic. Assume $S = 1.1200$ USD per EUR, $T = 0.5$ years, $r_d = 4.00\%$ and $r_f = 2.00\%$ per year. The European options exercise only at expiry. Premiums are paid today in USD. The quote convention is the unadjusted forward delta and delta-neutral ATM defined above. Take $A = 10.00\%$, $R = -1.60$ points and $B = +0.50$ points on the arithmetic smile basis. On the QuaRC slide's scale the butterfly would read +1.00 point.

The forward is $1.1200e^{0.01} = 1.13125619$ and one dollar payable at expiry is worth $e^{-0.02} = 0.98019867$ dollars today. The mean wing volatility is 10.50%. Half the risk reversal is −0.80 points, which puts the call at 9.70% and the put at 11.30%. As a check, their difference is −1.60 points and their average exceeds ATM by 0.50 points.

For the put, $\Phi(d_1)-1 = -0.25$ gives $d_1 = 0.674490$. Its six-month width is $w = 0.113\sqrt{(0.5)} = 0.079903$. The strike follows as $1.13125619\exp(0.079903^2/2 - 0.079903 \times 0.674490) = 1.07533$ to five decimals. Using $d_2 \simeq 0.594587$ in the put formula gives 0.01376326 dollars per euro.

Purchased option	Signed forward delta	Annualized implied volatility	Strike (USD/EUR)	Premium today (USD per EUR of notional)
25-delta EUR put	−0.25	11.30%	1.07532951	0.01376326
ATM EUR put	−0.50	10.00%	1.13408787	0.03272023
ATM EUR call	+0.50	10.00%	1.13408787	0.02994463
25-delta EUR call	+0.25	9.70%	1.18761103	0.01096582

Table 1. Author's calculations from the synthetic quotes above. The two ATM rows share one smile point. Values are rounded after pricing.

For an exporter expecting EUR 1 million, the 25-delta put costs approximately 13,763 USD today. If spot is 1.0400 at expiry, selling the euros under the option yields about 1,075,330 USD, or 35,330 USD more than selling them at spot, before accounting for the initial premium. If spot finishes above the strike, the exporter can sell at the prevailing rate. The different prices of the two ATM legs provide another check. Put-call parity gives $C-P = D_d(F-K_A) \simeq -0.00277560$ USD per EUR. Equal and opposite deltas at ATM can coexist with unequal premiums.

A second screen quotes the price of a strangle

Suppose the same six-month screen now labels the +0.50-point entry **market strangle**. Keep $A = 10.00\%$, $R = -1.60$ points, both rates and spot as before. The numerical input has stayed the same, but the dealer has specified a different constraint. We can measure the difference by asking what each interpretation charges for exactly the same put and call.

Establish the dollar target. Add 0.50 points to ATM to obtain a common volatility of 10.50%. With that volatility in the delta-to-strike calculation on both sides, the put strike is $K_P^m = 1.07896972$ and the call strike is $K_C^m = 1.19263273$. The quoted package costs $0.012751609+0.011837525 = 0.024589134$ USD per EUR of notional on each leg. For one million euros per leg, that is about 24,589 USD in total. These two strikes are now contract terms. Recalculating them every time a trial volatility changes would change the contract being priced.

Choose how to fill the gaps. For this demonstration take annualized volatility to be a quadratic in log strike relative to delta-neutral ATM. Define $x = \log(K/K_A)$ and set

$$s_b(K) = A + u_b x + v_b x^2.$$

The subscript b is a trial *arithmetic* butterfly in decimal volatility. For each trial, the put wing has volatility $A+b-R/2$ and the call wing has volatility $A+b+R/2$. Convert each to its own 25-delta strike. Denote the resulting log strikes by x_P, x_C and the corresponding volatilities by s_P, s_C. The curve has the correct ATM value because $x = 0$ there. To make it pass through both wings, compute

$$h_P = (s_P - A)/x_P, \qquad h_C = (s_C - A)/x_C, \qquad v_b = (h_C - h_P)/(x_C - x_P), \qquad u_b = h_P - v_b x_P.$$

These equations specify the entire reconstruction in the neighbourhood of the instruments. In particular they give a value at the frozen call strike, which in this example lies slightly beyond the smile's 25-delta call. This is a short extrapolation and its choice affects the result. Reiswich and Wystup (2010, equations 24-25, 31 and 34-37) give the market-price restriction and develop an interpolation in delta space. The log-strike polynomial here is the author's choice for a transparent calculation at a single expiry.

Reprice the same strikes. First try $b = 0.005$, as if the quoted spread were the arithmetic butterfly. Then adjust b until the reconstructed sum of premiums matches the market package. Table 2 shows how the two legs contribute.

Treatment at frozen strikes	Put volatility	Call volatility	Put premium	Call premium	Combined premium
Common-volatility quote	10.50000%	10.50000%	0.01275161	0.01183752	0.02458913
Arithmetic shortcut, b = 0.005	11.18702%	9.71089%	0.01448486	0.00991117	0.02439603
Fitted smile, b = 0.0054175272	11.21764%	9.75896%	0.01456304	0.01002609	0.02458913

Table 2. Synthetic market-strangle comparison. Premiums are USD per EUR of notional on each leg. Put and call strikes stay at 1.07896972 and 1.19263273 USD/EUR throughout. Totals use unrounded leg values.

Under the shortcut, the lower-strike put costs more than it does under the common-volatility quote, while the higher-strike call costs less. The reduction in the call price exceeds the rise in the put price. The net shortfall is 0.0001931072 USD per EUR, or **193.11 USD per EUR million**. Adding the two wing premiums from Table 1 would produce a different number again, because both strikes in that table differ from the frozen strikes.

The numerical fit solves one equation. Write $P(K,s)$ and $C(K,s)$ for the premiums above evaluated at strike K and volatility s. Subtract the unrounded market package price from $P(K_P{}^m,s_b(K_P{}^m))+C(K_C{}^m,s_b(K_C{}^m))$. This residual changes sign between b = 0 and b = 0.02. Halving that bracket 48 times gives **0.541753 volatility points** as the fitted arithmetic butterfly. This is a model-dependent conversion. At the fitted smile's own 25-delta strikes, 1.07514081 and 1.18787166, the volatilities are 11.341753% and 9.741753%. Their difference, call less put, recovers −1.600000 points. At the market package's strikes, the two different volatilities in the last row of Table 2 recover the quoted sum. The market quotation specifies only that sum, leaving its allocation across legs to the fitted smile.

There is a simple way to challenge the calculation. Set R = 0 while retaining the same strangle spread. At $b = M$ both wing volatilities become $A+M$. Their 25-delta strikes must then coincide with the frozen strikes, so any interpolant passing through the nodes must recover the package premium. The code returns b = 0.0050000000 in this case. A program that still shows a material discrepancy here is probably changing the strikes or mishandling the signed put delta.

What changes when the delta label changes?

Return to the arithmetic quotes in Table 1 and switch only the wing labels to **premium-unadjusted spot delta**. The volatilities, forward and discount factors stay fixed. Since $\Delta_S = e^{-r_f T}\Delta_F$, the old 25-forward-delta contracts carry absolute spot delta $0.25e^{-0.01}$ = 0.24751246. Relabelling those existing contracts leaves their premiums unchanged. To order a fresh 25-*spot*-delta contract, however, the required absolute forward delta is $0.25e^{0.01}$ = 0.25251254. Applying that target gives the following comparison (Reiswich and Wystup 2010, equations 5 and 7).

Wing at the same quoted volatility	Forward-delta strike	Spot-delta strike	Forward-label premium	Spot-label premium
EUR put, 11.30%	1.07532951	1.07600728	0.01376326	0.01394754
EUR call, 9.70%	1.18761103	1.18696886	0.01096582	0.01111054

Table 3. Synthetic six-month contracts with absolute quoted delta 0.25. Strikes and premiums are in USD per EUR.

Both new strikes move towards the forward. The exporter can sell euros at a slightly higher floor, at an additional upfront cost of about 184 USD on one million euros. The delta-neutral ATM strike is

unchanged in this experiment because multiplying both ATM deltas by the same positive discount factor leaves their sum zero. In the limiting case $r_f = 0$, the distinction between these two unadjusted delta conventions also disappears in the wings.

What the smile leaves open

Each row fixes a European premium today. Prices at intermediate strikes require interpolation. A proposed completion should at least give domestic call prices that decrease with strike, stay convex and respect their intrinsic and upper bounds. For example, a negative price for a portfolio long the two outside calls and short twice the middle call at equally spaced strikes would contradict its nonnegative expiry payoff. Checks on three nodes alone cannot establish those properties throughout both tails.

The distinction between a terminal value and a path can be seen directly from the payoff. Under deterministic rates, a continuum of calls satisfies $C(K) = D_d E^d[(S_T - K)^+]$, where E^d is expectation under the domestic risk-neutral pricing measure. If the terminal distribution has a continuous density, differentiating with respect to strike gives $-C'(K)/D_d = \Pr^d(S_T > K)$. A second derivative gives the density. This argument needs prices across strikes. Three quotes leave many possible terminal distributions, and risk-neutral probabilities incorporate the market price of risk. Estimating actual frequencies requires additional evidence.

Now add a clause cancelling the exporter's put if EUR/USD ever trades at or below 1.0200 before or at expiry, monitored continuously. Two exchange rates can both finish at 1.0400. One path bottoms at 1.0300, leaving the 35,330 USD payoff intact. Another touches 1.0100 and recovers, leaving the holder with zero. Even a fully specified terminal distribution cannot tell us how much pricing probability to assign to each route. The missing quantity is the joint behaviour of the running minimum and the terminal rate.

A model for tomorrow's risk reversal likewise needs the distribution of tomorrow's *conditional* option prices. One possible specification makes instantaneous volatility a deterministic function of time and spot. Another gives variance its own random shocks and sets their correlation with spot shocks. That correlation affects how a spot move changes the relative prices of the wings. Itkin (2017) goes further by allowing the correlation to evolve randomly and by including jumps. Prices for liquid barrier contracts can help discriminate between dynamics that give similar vanilla premiums, a use discussed by Kainth and Saravanamuttu (n.d., slides “Liquid Barrier Products” and “Single Barrier Vanilla Payoffs”). The monitoring schedule, barrier level and rebate terms must agree before comparing those prices. For the exporter, the vanilla quote prices an expiry floor. Making that floor conditional on the path introduces a separate valuation assumption.

Two exchange-rate models with the same expiry prices

To put a price on the missing information, consider a separate two-step market with zero interest rates in both currencies and an initial exchange rate of 1.00. At maturity the possible rates are 0.80, 1.00 and 1.20 with pricing probabilities $1/4, 1/2, 1/4$. In both models below the expected next rate at each node equals the current rate. This martingale condition is the zero-rate counterpart of pricing consistently with the forward.

In model I the rate stays at 1.00 at the intermediate observation. All the uncertainty resolves at maturity. In model II the rate first moves to 0.90 or 1.10 with equal probability. From 0.90 it finishes at 0.80 or 1.00, equally likely. From 1.10 it finishes at 1.00 or 1.20, also equally likely. The two trees assign identical probabilities to every terminal outcome. Hence every European call and put for this expiry, at *every* strike, has the same initial price in both trees. A put struck at 1.10, for instance, is worth $0.25(0.30) + 0.50(0.10) = 0.125$ per unit of foreign currency in either.

Now price a contract paying one domestic currency unit at expiry if the rate is at or below 0.95 at the intermediate observation or at maturity. In model I, only the terminal 0.80 outcome pays, giving a price of 0.25. In model II, the entire lower first branch has already touched, giving 0.50. The extra 0.25 comes

entirely from paths that touched 0.90 and recovered to 1.00. This original example compares a single complete expiry slice under two specified observation dates. Intermediate-maturity vanillas would distinguish these particular trees. The historical slides' discussion of double no-touch and single-barrier products asks the related practical question of which traded prices give evidence about dynamics. Itkin's later numerical experiments explicitly compare European, down-and-out and double no-touch contracts (2017, sections 3 and 7.1-7.4).

For use beyond this example, the six-month calculation needs the expiry and delivery dates, consistent discount factors and the premium settlement convention for the actual pair. Bid and offer quotes would give a range of admissible package prices. Fitting one midpoint to numerical precision quantifies the solver error in Table 2. It gives no estimate of that trading range, the uncertainty in the interpolant or the cost of hedging a barrier.

Reproducing the examples

The accompanying `example.py` uses the Python standard library. Run `python example.py` to reproduce Table 1. The script independently integrates each discounted payoff over a standard normal shock to the terminal log exchange rate, using 16,000 midpoint intervals on the exercise region within ten standard deviations. Each option uses its own quoted volatility for this price check. It also bumps the forward up and down by one part in 100,000 and divides the centred price change by the discount factor to check each signed forward delta. Absolute errors were below 5.2×10^{-10} USD per EUR for premiums and 6.9×10^{-10} for deltas, against tolerances of 10^{-8} for each.

Run `python market_strangle.py` in the same directory for Tables 2 and 3, the zero-reversal experiment and both short trees. The script checks the package residual again by numerical integration of both fitted legs and both reference legs. The independently integrated difference for the skewed case is about −0.00000000000283 USD per EUR. A useful spreadsheet check is to fix the two market strikes in separate cells before varying the arithmetic butterfly. At every trial the risk reversal at the moving wing strikes should remain −1.60 points, while the combined premium at the fixed strikes should change.

Historical source note

The archived PowerPoint names Dherminder Kainth and Nagulan Saravanamuttu and gives their affiliation as QuaRC, Royal Bank of Scotland. Its title slide supplies no date. The original address cited by Itkin (2017, bibliography) was `http://www.quarchome.org/fxskew2.ppt`. The file inspected here came from the 30 December 2010 capture of `http://www.quarchome.org/FXSkew2.ppt`. Its archive digest matches the 6 September 2015 capture of the lowercase address. Itkin lists 2016 for the presentation, but the same file was already archived in 2010. The presentation date and any formal edition number remain unverified.

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Author note

Chris Kozlowski is an independent writer on economics, data and decision-making. He writes for readers who want to follow a calculation and judge how its conclusions depend on the assumptions.